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Mean-field approach to the derivation of baryon superpotential from matrix model

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Abstract: We discuss how to obtain the superpotential of the baryons and mesons for SU(N) gauge theories with N flavour matter fields from matrix integral. We apply the mean-field approximation for the matrix integral. Assuming the planar limit of the self-consistency equation, we show that the result almost agrees with the field theoretical result.

Keywords: 1/N Expansion, Supersymmetry and Duality.

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Contents

1. Introduction

Recently, Dijkgraaf and Vafa [1, 2, 3] proposed a general prescription for computing effective glueball superpotentials via planar diagrams of matrix theories. Several tests have been achieved for this conjecture [4]–[11] for the fields in adjoint representations. A recent paper [12, 13] proved a correspondence for matter fields in the adjoint representation. A natural extension of the construction is to the theories with fields in the fundamental representations. The gauge theory effective potential for glueball superfield S is given by

$$W_{DV}(S,\Lambda,\lambda_a) = N_c S\left(-\log\left(\frac{S}{\Lambda^3}\right)+1\right) + N_c\frac{\partial\mathcal{F}_{\chi=2}(S,\lambda_a)}{\partial S} + \mathcal{F}_{\chi=1}(S,\lambda_a)\,. \qquad (1.1)$$

where $\mathcal{F}_{\chi=2}$ is the contribution from adjoint fields and $\mathcal{F}_{\chi=1}(S,\lambda_a)$ is the contribution from the fields in the fundamental representations. The programm was successfully used to compare matrix model results with known superpotentials of meson fields [14]–[21].

The advantage the matrix theory should be that we can prove some basic hypothesis or conjectures which cannot be provided by the standard field theoretical methods. In fact, a derivation of the non-renormalization theorem was performed in [22].

One of the biggest problems is the inclusion of the baryonic fields. Matrix theory may be useful to get insights for the baryons. This problem was treated in a recent papers [23, 24]. The paper [23] showed that the first order calculation in the perturbative technique agrees with the field theoretical result. In [24], they showed that for SU(2) gauge fields with 2 flavor models, we can obtain superpotential for baryonic fields. In this model, the baryons can be expressed by bilinear fields and the matrix integration can be performed. However, for SU(N), $N > 2$, the baryons are composed of more than trilinear fields so that we cannot easily perform the matrix integrations.

In this paper, we consider the theories with SU(N) gauge fields with N flavour. The quantum moduli space is conjectured to be parametrized by meson fields and baryons by [26, 27, 28]

$$\det M - B\tilde{B} = \Lambda^{2N} . \tag{1.2}$$

Our aim of this paper is the derivation of this constraint from the matrix model. Since baryonic perturbation is complicated, we will use the mean-field method to estimate the superpotential, which is related to the free energy of the matrix model. It will turn out that the self-consistency equation for the mean field depends on N. We will therefore assume a planar limit of the equation. With this assumption, we will show that we can get a potential which almost agrees with the expected results. Before doing this, we will first treat the field theoretical result which should be compared to the matrix theory result.

2. Effective superpotentials for $N_c = N_f$

We will first argue how to obtain effective superpotential from the field theory. We consider SU(N) gauge theories with N quark flavors Q_a^i and $\tilde{Q}_i^a$. We diagonalize the mass matrix and consider the tree level superpotential

$$W_{\text{tree}}\left(Q, \tilde{Q}\right) = m_i \tilde{Q}_i^a Q_a^i + b \det Q + \tilde{b} \det \tilde{Q} , \tag{2.1}$$

which can be written in terms of meson operators and baryon operators

$$M_i^j = Q_i^a \tilde{Q}_a^j , \qquad B = \det Q , \qquad \tilde{B} = \det \tilde{Q} \tag{2.2}$$

as

$$W_{\text{tree}}\left(X_i, B, \tilde{B}\right) = m_i X_i + bB + \tilde{b}\tilde{B} , \tag{2.3}$$

where we have defined $X_i = M_i^i$. In order to obtain the non perturbative part of the effective superpotential, we start with the Veneziano-Yankielowicz superpotential [25].

$$W = NS - S \ln \frac{S^N}{\tilde{\Lambda}^{3N}} . \tag{2.4}$$

For N-flavour, we assume the matching relation $\tilde{\Lambda}^{3N} = m_1 m_2 \cdots m_N \Lambda^{2N}$. When we integrated in X_i fields as $X_i = \partial_{m_i} W$, we get the following superpotential

$$W = S \ln \frac{\Lambda^{2N}}{X_1 X_2 \cdots X_N} + m_i X^i , \tag{2.5}$$

which can be promoted to

$$W = S \ln \frac{\Lambda^{2N}}{\det M} + m_i X^i . \tag{2.6}$$

If we include the baryon fields, it is natural to expect that the effective potential is given by

$$W = S \ln \frac{\Lambda^{2N}}{-B\tilde{B} + \det M} + bB + \tilde{b}\tilde{B} + m_i X^i . \tag{2.7}$$

The equation $\partial_S W = 0$ implies the constraint

$$\det M - B\tilde{B} = \Lambda^{2N}, \tag{2.8}$$

which is a desired result [26, 27, 28]. We assume that this potential is exact although the proof of the non-renormalization theorem of baryoionic perturbation may be very subtle.

To compare this result with the superpotential from the matrix models, we are going to integrated out the fields $B, \tilde{B}$ and M_i^j. Let us first denote

$$\Delta = \det M - B\tilde{B} \tag{2.9}$$

The equation $\partial_B W = 0, \partial_{\tilde{B}} W = 0$ implies

$$B = -\frac{\Delta \tilde{b}}{S}, \qquad \tilde{B} = -\frac{\Delta b}{S}. \tag{2.10}$$

The variation with respect to M_i^j leads to

$$\begin{aligned} M_i^j &= 0, \quad (i \neq j) \\ m_i X_i &= \left(\frac{\Delta}{S} m_1 m_2 \cdots m_N\right)^{\frac{1}{N-1}}. \end{aligned} \tag{2.11}$$

From these relations, we find that Δ satisfies an equation

$$\Delta = X_1 X_2 \cdots X_N - B\tilde{B} = \left(\frac{\Delta}{S}\right)^{\frac{N}{N-1}} - \left(\frac{\Delta}{S}\right)^2 b\tilde{b}. \tag{2.12}$$

when $b = 0$, this equation has a solution $\Delta = S^N / m_1 m_2 \cdots m_N$. Therefore, we define a new variable by $\Delta = S^N / m_1 m_2 \cdots m_N \tilde{\Delta}$. Then the equation for $\tilde{\Delta}$ is given by

$$\tilde{\Delta} = \left(1 + \alpha \tilde{\Delta}\right)^{N-1}, \tag{2.13}$$

where α is defined as

$$\alpha = \frac{S^{N-2} b\tilde{b}}{m_1 m_2 \cdots m_N}. \tag{2.14}$$

Substituting these variables in (2.7), we find that the effective superpotential can be written in the form

$$W = NS\left(1 - \ln S/\Lambda^3\right) + \mathcal{F}, \tag{2.15}$$

where $\mathcal{F}$ is

$$\mathcal{F} = S \ln \frac{m_1 m_2 \cdots m_N}{\Lambda^N} - S \ln \tilde{\Delta} + (N-2)\alpha S \tilde{\Delta}. \tag{2.16}$$

This is the final form of the superpotential which we want to compare in the next section. Note also that the superpotential (2.16) agrees with the one obtained in ref [23] although the derivation seems different.

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3. Mean-field approximation for matrix model

3.1 Mean field approach

Before discussing the model for baryonic perturbation, we will apply the mean-field method to the known models. The aim of doing this is to get a support for the validity of the mean-field method. The tree level superpotential we treat here is obtained by the deformation from N=2 SQCD theory with fundamental flavors Q_i^a and $\tilde{Q}_i^a$ by adding a mass M for the adjoint scalar Φ_a^b in N=2 vector multiplet

$$W_{\rm tree}\left(\Phi, Q, \tilde{Q}\right) = \frac{1}{2}M{\rm Tr}\Phi^2 + g\tilde{Q}_i^a\Phi_a^bQ_b^i + m\tilde{Q}_i^aQ_a^i, \tag{3.1}$$

where we have set the mass of the fundamental flavors to be $m_1 = m_2 = \ldots = m_{N_f} = m$ for simplicity. We consider the following free energy

$$e^{-\frac{N}{S}\mathcal{F}} = \left(\frac{N\Lambda}{S}\right)^{N_f}\int d\Phi dQ d\tilde{Q} e^{-\frac{N}{S}W_{\rm tree}(\Phi,Q,\tilde{Q})}, \tag{3.2}$$

Since the integral for the adjoint fields Φ_a^b is gaussian, we can integrate out the fields and obtain

$$e^{-\frac{N}{S}\mathcal{F}} = \left(\frac{N\Lambda}{S}\right)^{N_f}\int d\Phi dQ d\tilde{Q} e^{-\frac{N}{S}\left[m\tilde{Q}_i^aQ_a^i - \frac{g^2}{2M}\tilde{Q}_i^aQ_a^j\tilde{Q}_j^bQ_b^i\right]}, \tag{3.3}$$

We re-scale the variables as

$$Q_i^a \to \left(\frac{N}{S}m\right)^{-1/2} Q_i^a\,, \qquad \tilde{Q}_a^i \to \left(\frac{N}{S}m\right)^{-1/2}\tilde{Q}_a^i\,, \tag{3.4}$$

to get

$$e^{-\frac{N}{S}\mathcal{F}} = \left(\frac{\Lambda}{m}\right)^{NN_f}\int d\Phi dQ d\tilde{Q} e^{-W}\,, \tag{3.5}$$

where

$$W = \tilde{Q}_i^aQ_a^i - \frac{\beta S}{2N}\tilde{Q}_i^aQ_a^j\tilde{Q}_j^bQ_b^i\,, \qquad \beta = \frac{g^2}{mM^2}\,. \tag{3.6}$$

We will write the correlation function $\langle\tilde{Q}_i^aQ_b^j\rangle$ as

$$\langle\tilde{Q}_i^aQ_b^j\rangle = \Delta'\delta_i^j\delta_b^a\,, \tag{3.7}$$

Then the above tree level potential$W$ can be written in the form

$$\begin{aligned} W &= \tilde{Q}_i^aQ_a^i - \beta S\Delta'\tilde{Q}_i^aQ_a^i + NN_f\frac{\beta S}{2}\Delta'^2 - \\ &-\frac{\beta S}{2N}\sum_{i,j=1}^{N_f}\left\{\left[\sum_{a=1}^{N}\left(\tilde{Q}_i^aQ_a^j - \Delta'\delta_i^j\right)\right]\left[\sum_{b=1}^{N}\left(\tilde{Q}_j^bQ_b^i - \Delta'\delta_j^i\right)\right]\right\}. \end{aligned} \tag{3.8}$$

As a mean field theory, we neglect the contribution of the last term. Namely, we consider the following free energy:

$$e^{-\frac{N}{S}\mathcal{F}} = \left(\frac{\Lambda}{m}\right)^{NN_f}\int d\Phi dQ d\tilde{Q} e^{-[\tilde{Q}_i^aQ_a^i - \beta S\Delta'\tilde{Q}_i^aQ_a^i + NN_f\frac{\beta S}{2}\Delta'^2]}\,. \tag{3.9}$$

We can compute the correlation function $\langle \tilde{Q}^a_i Q^j_b \rangle = \Delta' \delta^j_i \delta^a_b$ and get a consistency equation for Δ':

$$\frac{1}{1 - \beta S \Delta'} = \Delta' \,. \tag{3.10}$$

This equation can be easily solved by

$$\Delta' = \frac{1 - \sqrt{1 - 4\beta S}}{2\beta S} = \frac{2}{1 + \sqrt{1 - 4\beta S}} \,. \tag{3.11}$$

Performing the gaussian integral in (3.9), we get a free energy

$$\begin{aligned} \mathcal{F} &= N_f S \left[\ln \frac{\Lambda}{m} - \ln \Delta' + \frac{\beta S}{2} \Delta'^2) \right] \\ &= N_f S \left[\ln \frac{\Lambda}{m} - \frac{1}{2} - \frac{1}{4\beta S} \left(1 - \sqrt{1 - 4\beta S} \right) + \ln \frac{1 + \sqrt{1 - 4\beta S}}{2} \right] \end{aligned} \tag{3.12}$$

which is exactly the result obtained by summing up the ladder diagrams [14].

In general, we may have N dependence in the self-consistency equation (3.10). Then, we should take the planar limit where N dependence disappears. Then we expect that the result reflects the contribution from the planar diagrams.

3.2 Effective superpotential for baryons from matrix theory

Let us apply the mean-field technique to the baryonic perturbation. We consider the free energy

$$e^{-\frac{N}{S}\mathcal{F}} = \left(\frac{N\Lambda}{S} \right)^N \int \prod_{i,a=1}^{N} dQ^a_i d\tilde{Q}^i_a e^{-\frac{N}{S} W_{\rm tree}(Q,\tilde{Q})} \,, \tag{3.13}$$

which is in our case,

$$\begin{aligned} e^{-\frac{N}{S}\mathcal{F}} &= \left(\frac{N\Lambda}{S} \right)^N \times \\ &\times \int \prod_{i,a=1}^{N} dQ^a_i d\tilde{Q}^i_a e^{-\frac{N}{S}(m_i \tilde{Q}^a_i Q^i_a + b\epsilon^{a_1 a_2 \cdots a_N} Q^1_{a_1} Q^2_{a_2} \cdots Q^N_{a_N} + \tilde{b}\epsilon_{b_1 b_2 \cdots b_N} \tilde{Q}^{b_1}_1 \tilde{Q}^{b_2}_2 \cdots \tilde{Q}^{b_N}_N)} \,, \end{aligned} \tag{3.14}$$

The superpotential can be derived by the free energy $\mathcal{F}$ as

$$W = NS \left(1 - \ln \frac{S}{\Lambda^3} \right) + \mathcal{F} \,. \tag{3.15}$$

The first term of the superpotential is the Veneziano-Yankielowicz superpotential which can be obtained either from the volume of the gauge groups [12] or gaussian integral measures [13]. We will first re-scale the variables

$$Q^a_i \to \left(\frac{N}{S} m_i \right)^{-1/2} Q^a_i \,, \qquad \tilde{Q}^i_a \to \left(\frac{N}{S} m_i \right)^{-1/2} \tilde{Q}^i_a \,. \tag{3.16}$$

Then the matrix integral can be written as

$$e^{-\frac{N}{S}\mathcal{F}} = \left(\frac{\Lambda^N}{m_1 m_2 \cdots m_n} \right)^N \int \prod_{i,a=1}^{N} dQ^a_i d\tilde{Q}^i_a e^{-W} \,, \tag{3.17}$$

where

$$W = \tilde{Q}_i^a Q_a^i - \left[\left(\frac{S}{N} \right)^{N-2} \frac{1}{m_1 m_2 \cdots m_n} \right]^{1/2} \times$$
$$\times \left(b\epsilon^{a_1 a_2 \cdots a_N} Q_{a_1}^1 Q_{a_2}^2 \cdots Q_{a_N}^N + \tilde{b}\epsilon_{b_1 b_2 \cdots b_N} \tilde{Q}_1^{b_1} \tilde{Q}_2^{b_2} \cdots \tilde{Q}_N^{b_N} \right). \quad (3.18)$$

Integrating out the fields Q_a^N and $\tilde{Q}_N^a$, we get

$$e^{-\frac{N}{S}\mathcal{F}} = \left(\frac{\Lambda^N}{m_1 m_2 \cdots m_n} \right)^N \int \prod_{i=1}^{N-1} \prod_{a=1}^{N} dQ_i^a d\tilde{Q}_a^i e^{-\tilde{W}} \quad (3.19)$$

where $\tilde{W}$ is given by

$$\tilde{W} = -\tilde{Q}_i^a Q_a^i + \frac{\alpha}{N^{N-2}} \epsilon^{a_1 a_2 \cdots a_N} \epsilon_{b_1 b_2 \cdots a_N} Q_{a_1}^1 Q_{a_2}^2 \cdots Q_{a_{N-1}}^{N-1} \tilde{Q}_1^{b_1} \tilde{Q}_2^{b_2} \cdots \tilde{Q}_{N-1}^{b_{N-1}}, \quad (3.20)$$

and the parameter α is defined in (2.14). To estimate the free energy, we apply the mean-field approximation. We put the correlation functions

$$\left\langle \epsilon^{a a_1 a_2 \cdots a_{N-1} a_N} \epsilon_{b_1 b_2 \cdots b_{N-1} a_n} \prod_{k \neq i}^{N-1} Q_{a_i}^k \tilde{Q}_k^{b_i} \right\rangle$$

in the form

$$\left\langle \epsilon^{a a_1 a_2 \cdots a_{N-1} a_N} \epsilon_{b_1 b_2 \cdots b_{N-1} a_n} \prod_{k \neq i}^{N-1} Q_{a_i}^k \tilde{Q}_k^{b_i} \right\rangle = (N-1) N^{N-2} \Delta' \delta_{a_i}^{b_i}. \quad (3.21)$$

Then we can rewrite $\tilde{W}$ as

$$\begin{aligned} \tilde{W} = & -\tilde{Q}_i^a Q_a^i + \alpha\Delta' \tilde{Q}_i^a Q_a^i - \alpha\Delta' \langle Q_a^i \tilde{Q}_i^a \rangle + \\ & + \sum_{i=1}^{N-1} \frac{\alpha \epsilon^{a_1 a_2 \cdots a_N} \epsilon_{b_1 b_2 \cdots a_N}}{(N-1) N^{N-2}} \prod_{k \neq i}^{N-1} Q_{a_i}^k \tilde{Q}_k^{b_i} \langle Q_{a_i}^i \tilde{Q}_i^{b_i} \rangle + \\ & + \alpha \sum_{i=1}^{N-1} \left(\frac{\epsilon^{a_1 a_2 \cdots a_N} \epsilon_{b_1 b_2 \cdots a_N}}{(N-1) N^{N-2}} \prod_{k \neq i}^{N-1} Q_{a_i}^k \tilde{Q}_k^{b_i} - \Delta' \delta_{b_i}^{a_i} \right) \left(Q_{a_i}^i \tilde{Q}_i^{b_i} - \langle Q_{a_i}^i \tilde{Q}_i^{b_i} \rangle \right). \end{aligned} \quad (3.22)$$

As a mean field evaluation, we neglect the contribution of the last term of equation (3.22). We also omit the contribution of the term

$$A \sum_{i=1}^{N-1} \frac{\alpha \epsilon^{a_1 a_2 \cdots a_N} \epsilon_{b_1 b_2 \cdots a_N}}{(N-1) N^{N-2}} \prod_{k \neq i}^{N-1} Q_{a_i}^k \tilde{Q}_k^{b_i} \langle Q_{a_i}^i \tilde{Q}_i^{b_i} \rangle. \quad (3.23)$$

Note that the omitting of this term is not completely valid for $N = 2$ and $N = 3$. For $N = 2$, this term is a constant and should be included in the free energy. This case, the integral in (3.19) can be performed without any approximation. We can easily see that the resulting expression of free energy agrees with the field theoretical results(2.16) for $N = 2$.

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For $N=3$, (3.23) is bilinear in fields and we should prepare more generic ansatz for the correlator

$$\left\langle \epsilon^{aa_1a_2\cdots a_{N-1}a_N}\epsilon_{b_1b_2\cdots b_{N-1}a_n}\prod_{k\neq i}^{N-1} Q^k_{a_i}\prod_{k\neq j}\tilde{Q}^{b_i}_k\right\rangle = (N-1)N^{N-2}\Delta'^{b_j,i}_{a_i,j}\,, \tag{3.24}$$

to get a proper consistency equation. This ansatz is generic, and we should use for generic value of N. However, the solution of the self-consistency equation seems very difficult to be solved. We therefore, assume $N\rangle 3$ for later calculation and negrect these terms. Then we find that the remaining integrations are just gaussian integrals so that we can get a self-consistent equation for Δ' via the equation (3.21).

The gap equation of this model can be easily obtained as

$$(N-1)!\left(\frac{1}{1-\alpha\Delta'}\right)^{N-2} = (N-1)N^{N-2}\Delta'\,, \tag{3.25}$$

which can be written as

$$\left(1-\frac{2}{N}\right)\left(1-\frac{3}{N}\right)\cdots\left(1-\frac{N-1}{N}\right)\left(\frac{1}{1-\alpha\Delta'}\right)^{N-2} = \Delta'\,. \tag{3.26}$$

If we can take a very "naive" planar limit, we obtain the gap equation

$$\left(\frac{1}{1-\alpha\Delta'}\right)^{N-2} = \Delta'\,. \tag{3.27}$$

We should admit that this "planar" limit is too subtle. This may be originated from a simple ansatz for the correlation functions (3.21). As a diagrammatic calculation, we should select only a small part of the diagrams to get this form of correlation functions. Therefore, we should use more generic ansatz (3.24) to perform a more rigorous analysis. However, the gap equation becomes very complicated. A partial evidence of the appearance of the gap equation (3.27) is the following. Instead of considering the more general ansatz, we enhance the number of diagrams we should correct by changing the correlation functions. Namely, we enhance the number by changing the equation as

$$N\left\langle \prod_{k\neq i}^{N-1}\left(\sum_{a_j} Q^k_{a_j}\tilde{Q}^{a_j}_k\right)\right\rangle = (N-1)N^{N-2}\Delta'\delta^{b_i}_{a_i}\,. \tag{3.28}$$

Since the correlation function can $\langle Q^a_i\tilde{Q}^j_b\rangle$ can be given by

$$\langle Q^a_i\tilde{Q}^j_b\rangle = \frac{\delta_{ij}\delta_{ab}}{1-\alpha\Delta'}\,, \tag{3.29}$$

we get a "gap equation" in the form

$$\left(\frac{1}{1-\alpha\Delta'}\right)^{N-2} = \left(1-\frac{1}{N}\right)\Delta'\,. \tag{3.30}$$

As a "planar" equation, this equation leads to (3.27).

In this paper, we assume the "planar" gap equation is given by (3.27). The reason we insist on the appearance of the gap equation will be given bellow. When we change variables to $\tilde{\Delta}$ by

$$\frac{1}{1-\alpha\Delta'} = 1+\alpha\tilde{\Delta}\,, \tag{3.31}$$

we can easily find that equation (3.27) leads to

$$\tilde{\Delta} = \left(1+\alpha\tilde{\Delta}\right)^{N-1}\,, \tag{3.32}$$

which is exactly the equation (2.13). This strongly suggests that the exact planer limit of the gap equation is given by (3.27).

Let us evaluate the free energy within our approximation. Performing the gaussian integrations, we can evaluate the free energy as

$$\mathcal{F} = S\ln\frac{m_1 m_2\cdots m_N}{\Lambda^N} + S(N-1)\ln(1-\alpha\Delta') + \alpha S(N-1)\Delta'\frac{1}{1-\alpha\Delta'}\,. \tag{3.33}$$

We rewrite the free energy in terms of $\tilde{\Delta}$ by the identification (3.31) to obtain

$$\mathcal{F} = S\ln\frac{m_1 m_2\cdots m_N}{\Lambda^N} - S\ln\tilde{\Delta} + \alpha S(N-1)\tilde{\Delta}\,. \tag{3.34}$$

This is the final form of the free energy which should be compared to the field theoretical result(2.16):

$$\mathcal{F} = S\ln\frac{m_1 m_2\cdots m_N}{\Lambda^N} - S\ln\tilde{\Delta} + \alpha S(N-2)\tilde{\Delta}\,. \tag{3.35}$$

The difference of the last term may be related to our subtle planar limit.

4. Discussions

We have discussed a method to obtain the effective superpotential for baryons and mesons by using the mean-field approximation. We have derived a result which is almost identicat to the field theoretical result. However, we have assumed "planar limit" of the self-consistency equation for the derivation. We could not prove the conjecture since we have used a simple ansatz for the correlation functions. We should use more rigorous ansatz to get an improved result. It seems interesting that the resulting expression from the field theory is of the form which may be fully obtained by the mean-field method. Our result may indicates the existence of some simple derivation of the superpotential for baryonic fields.

Another Interesting application of the mean-field method is for the gauge theories with $N_f > N_c$. In this case, the most interesting aspect is the appearance of Seiberg duality which has been treated in the framework of the matrix models in [29, 30, 21] However, the baryonic degree of freedom will be required for the complete derivation of the duality. Mean-field approach may be useful to get the duality map of the fields identification.

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